Specific Formulas

Integrals of Elementary Functions

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} \qquad (a \neq -1)$$

$$\int x^{-1} \, dx = \ln|x|$$

$$\int e^x \, dx = e^x$$

$$\int \ln x \, dx = x \ln x - x$$

$$\int \sin x \, dx = -\cos x$$

$$\int \cos x \, dx = \sin x$$

$$\int \tan x \, dx = \ln|\sec x|$$

$$\int \cot x \, dx = -\ln|\csc x| = \ln|\sin x|$$

$$\int \sec x \, dx = \ln|\sec x + \tan x|$$

$$\int \csc x \, dx = -\ln|\csc x + \cot x| = \ln|\csc x - \cot x|$$

$\int \frac{\sin 2x}{x^2} dx = 2\ln x$

$\frac{\sin 2x}{x^2} = \frac{2}{x} - \frac{2^3}{3!}\cancel{x} + \frac{2^5}{5!}x^3 - \frac{2^7}{7!}x^5 + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$R = \frac{\frac{a}{2} + \omega t}{k}$

$\sin[\varrho(kR - \omega t)] = \sin\varrho$; $\frac{\sin\varrho}{\varrho^2} = \frac{k^2 \sin\varrho}{\frac{a^2}{4} + a\omega t + \omega^2 t^2}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

Some Other Elementary Integrals

$$\int \sec^2 x \; dx = \tan x$$

$$\int \csc^2 x \; dx = -\cot x$$

$$\int \sec x \tan x = \sec x$$

$$\int \csc x \cot x = -\csc x$$

$$\int \sinh x \; dx = \cosh x$$

$$\int \cosh x \; dx = \sinh x$$

$$\int \tanh x \; dx = \ln(\cosh x)$$

$$\int \coth x \; dx = \ln|\sinh x|$$

$$\int \operatorname{sech}^2 x \; dx = \tanh x$$

$$\int \operatorname{csch}^2 x \; dx = -\coth x$$

$$\int \operatorname{sech} x \tanh x \; dx = -\operatorname{sech} x$$

$$\int \operatorname{csch} x \coth x \; dx = -\operatorname{csch} x$$

If the following are to be memorized, the second alternative is preferred in both cases.

$$\int \sin^2 x\,dx = \begin{cases} \frac{1}{2}x - \frac{1}{4}\sin(2x) \\ \frac{1}{2}(x - \sin x \cos x) \end{cases}$$

$$\int \cos^2 x\,dx = \begin{cases} \frac{1}{2}x + \frac{1}{4}\sin(2x) \\ \frac{1}{2}(x + \sin x \cos x) \end{cases}$$

$$\int \frac{\sin(2x)}{x^2}\,dx = \int \frac{-4(\int \sin^2 x\,dx) + 2x}{x^2}$$

$$= -4\int \frac{\int \sin^2 x\,dx}{x^2} + 2\int \frac{1}{x}\,dx$$

$$= -2\int \frac{\int \sin^2 x\,dx}{\int x\,dx} + 2\int \frac{1}{x}\,dx$$

Selected Algebraic Forms

There is little point in memorizing the following, since they are all easily derived by trig substitution. However, the next to last one occurs so often in the method of partial fractions that it is certainly worth knowing, at least in the special case $a = 1$.

$$\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \sin^{-1} x \\ -\cos^{-1} x \end{cases}$$

$$\int \frac{dx}{1+x^2} = \begin{cases} \tan^{-1} x \\ -\cot^{-1} x \end{cases}$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \begin{cases} \sec^{-1} x \\ -\csc^{-1} x \end{cases}$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x$$

$$\int \frac{dx}{1-x^2} = \tanh^{-1} x$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} \qquad (a > 0)$$

$u = \sin(2x)$

$dv = \frac{1}{x^2}\,dx$

$v = -\frac{1}{x}$

$u = \cos(2x)$

$dv = \frac{1}{x}\,dx$

$v = \ln x$

$u = \cos(2x)$

$dv = \frac{1}{x^2}\,dx$

$$-\frac{\sin(2x)}{x} - \int -\frac{2\cos(2x)}{x}\,dx$$

$$= -\frac{\sin(2x)}{x} + 2\int \frac{\cos(2x)}{x}\,dx$$

$$= -\frac{\sin(2x)}{x} + 2\cos(2x)\ln x - 2\int \ln x\,(-2)\sin(2x)\,dx$$

$$= -\frac{\sin(2x)}{x} + 2\cos(2x)\ln x + 4\int \sin(2x)\ln x\,dx$$

$$\int \frac{\cos(2x)}{x^2}\,dx = -\frac{\cos(2x)}{x} - \int -\frac{1}{x}(-2)\sin(2x)\,dx$$

$$= -\frac{\cos(2x)}{x} - 2\int \frac{\sin(2x)}{x}\,dx$$

$$= -\frac{\cos(2x)}{x} - 2\sin(2x)\ln x + 2\int \ln x \cdot 2\cos(2x)\,dx$$

$$= -\frac{\cos(2x)}{x} - 2\sin(2x)\ln x + 4\int \cos(2x)\ln x\,dx$$

$J \quad \sqrt{a^2 - x^2} \qquad\qquad a$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \qquad (a > 0)$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} \qquad (a > 0)$$