

$$A \times (B \times C) = (C \times B) \times A = B(A \cdot C) - C(A \cdot B)$$

$$\nabla(uv) = u\nabla v + v\nabla u$$

$$\nabla(A \cdot B) = A \times (\nabla \times B) + (A \cdot \nabla)B + B \times (\nabla \times A) + (B \cdot \nabla)A$$

$$\nabla \cdot uA = u\nabla \cdot A + A \cdot \nabla u$$

$$\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$$

$$\nabla \times (uA) = u\nabla \times A - A \times \nabla u$$

$$\nabla \times (A \times B) = (B \cdot \nabla)A + A(\nabla \cdot B) - (A \cdot \nabla)B - B(\nabla \cdot A)$$

$$\overline{(\nabla \cdot A)B} = (A \cdot \nabla)B + B(\nabla \cdot A)$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - (\nabla \cdot \nabla)A$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

$$\nabla(fg) = f\nabla g + g\nabla f \quad \text{or, equivalently, } \text{grad}(fg) = f \text{ grad } g + g \text{ grad } f$$

$$\nabla(\mathbf{v} \cdot \mathbf{w}) = (\mathbf{v} \cdot \nabla)\mathbf{w} + (\mathbf{w} \cdot \nabla)\mathbf{v} + \mathbf{v} \times (\nabla \times \mathbf{w}) + \mathbf{w} \times (\nabla \times \mathbf{v})$$

$$\nabla \cdot (f\mathbf{v}) = f \nabla \cdot \mathbf{v} + \nabla f \cdot \mathbf{v} \quad \text{or, equivalently, } \text{div}(f\mathbf{v}) = f \text{ div } \mathbf{v} + \text{grad } f \cdot \mathbf{v}$$

$$\nabla \times (f\mathbf{v}) = f \nabla \times \mathbf{v} + \nabla f \times \mathbf{v} \quad \text{or, equivalently, } \text{curl}(f\mathbf{v}) = f \text{ curl } \mathbf{v} + \text{grad } f \times \mathbf{v}$$

$$\nabla \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot (\nabla \times \mathbf{v}) - \mathbf{v} \cdot (\nabla \times \mathbf{w})$$

$$\nabla \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\nabla \cdot \mathbf{w}) - \mathbf{w}(\nabla \cdot \mathbf{v}) + (\mathbf{w} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{w}$$

$$\nabla \cdot (\nabla \times \mathbf{B}) = \text{curl } \nabla \cdot \mathbf{B} - \nabla(\nabla \cdot \mathbf{B})$$

$\begin{matrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \end{matrix}$   
 $\partial_x \partial_x + \partial_y \partial_y + \partial_z \partial_z$

1.

Green's identities:

1.  $\int_{\Omega} f \nabla^2 g \, d^{\pi}x = \int_{\partial \Omega} f \mathbf{n} \cdot \nabla g \, d^{\pi-1}x - \int_{\Omega} \nabla f \cdot \nabla g \, d^{\pi}x$
2.  $\int_{\Omega} f \nabla^2 g - g \nabla^2 f \, d^{\pi}x = \int_{\partial \Omega} f \mathbf{n} \cdot \nabla g - g \mathbf{n} \cdot \nabla f \, d^{\pi-1}x$

Gauss's divergence theorem:

$$\int_{\Omega} \nabla \cdot \mathbf{F} \, d^{\pi}x = \int_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} \, d^{\pi-1}x$$

Stokes's theorem:

$$\int_{\Sigma} \nabla \times \mathbf{F} \cdot \mathbf{n} \, d^{\pi-1}x = \int_{\partial \Sigma} \mathbf{F} \cdot d\mathbf{l}$$