Electrodynamics with the scalar field

K.J. van Vlaenderen
koen@truth.myweb.nl

A. Waser
aw@aw-verlag.ch

October 27, 2001

Abstract

The theory of electrodynamics can be cast into biquaterion form. Usually Maxwells equations are invariant with respect to a gauge transformation of the potentials and one can choose freely a gauge condition. For instance, the Lorentz gauge condition yields the potential Lorenz inhomogeneous wave equations. It is possible to introduce a scalar field in the Maxwell equations such that the generalised Maxwell theory, expressed in terms of the potentials, automatically satisfy the Lorenz inhomogeneous wave equations, without any gauge condition. This theory of electrodynamics is no longer gauge invariant with respect to a transformation of the potentials: it is electrodynamics with broken gauge symmetry. The appearence of the extra scalar field terms can be described as a conditional *current* regauge that does not violate the conservation of charge, and it has several consequences:
- the prediction of a longitudinal electroscalar wave (LES wave) in vacuum.
- superluminal wave solutions, and possibly classical theory about photon tunneling.
- a generalised Lorentz force expression that contains an extra scalar term.
- generalised energy and momentum theorems, with an extra power flow term associated with LES waves.
A charge density wave that only induces a scalar field is possible in this theory.

1 Introduction: quaternions and biquaternions

The theory of electrodynamics can be formulated very efficiently in *biquaternion* form . Hamilton's quaternions [1] were used also by J.C. Maxwell in his Treatise on Electricity and Magnetism [2] [3]. A. Waser has used the biquaternion form also in [4] . The quaternion is very suitable to express the typical 4-qualities in physics, such as 4-position, 4-speed, 4-momentum, 4-force, 4-potential and 4-current [5]. A quaternion is defined as next:

$$\mathrm{X} = \mathrm{x}_0 + i\mathrm{x}_1 + j\mathrm{x}_2 + k\mathrm{x}_3 \tag{1}$$

where i, j, k are hypercomplex roots of -1, and $\mathrm{x}_0, \mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3$ are real numbers.

$$ii = jj = kk = -1 \qquad ij = -k \qquad jk = -i \qquad ij = -k \tag{2}$$

The scalar part of the quaternion is represented by x_0, while the vector part is represented by $\mathrm{x}_1, \mathrm{x}_2$ and x_3. If we define $\vec{\mathrm{x}} = (\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$ and $\vec{i} = (i, j, k)$ then we can notate a quaternion in a short scalar vector form by the use of the internal vector product:

$$\mathrm{X} = \mathrm{x}_0 + \vec{i} \cdot \vec{\mathrm{x}} \tag{3}$$

This notation makes the use of prefixes S or V, for indicating the scalar or vector part of a quaternion, redundant. Now the internal and external vector products and the vector itself can be used in quaternion equations. The quaternion sum and product are as follows:

$$\mathrm{X} + \mathrm{Y} = (\mathrm{x}_0 + \mathrm{y}_0) + \vec{i} \cdot (\vec{\mathrm{x}} + \vec{\mathrm{y}}) \tag{4}$$

$$\mathrm{XY} = (\mathrm{x}_0\mathrm{y}_0 - \vec{\mathrm{x}} \cdot \vec{\mathrm{y}}) + \vec{i} \cdot (\mathrm{x}_0\vec{\mathrm{y}} + \mathrm{y}_0\vec{\mathrm{x}} + \vec{\mathrm{x}} \times \vec{\mathrm{y}}) \tag{5}$$

This product is the consequence of the product rules of the Hamiltonian numbers i, j, k as defined in (2). Note that $\mathrm{XY} \neq \mathrm{YX}$, because $\vec{\mathrm{x}} \times \vec{\mathrm{y}} = -\vec{\mathrm{y}} \times \vec{\mathrm{x}}$. The quaternion product is associative and distributive:

$$\mathrm{X}(\mathrm{YZ}) = (\mathrm{XY})\mathrm{Z} \qquad \mathrm{X}(\mathrm{Y} + \mathrm{Z}) = \mathrm{XY} + \mathrm{XZ} \tag{6}$$

The conjugate and the length of a quaternion are defined as follows:

$$\mathrm{X}^* = \mathrm{x}_0 - \vec{i} \cdot \vec{\mathrm{x}} \tag{7}$$

$$|\mathrm{X}| = \sqrt{\mathrm{XX}^*} = \sqrt{\mathrm{x}_0^2 + \vec{\mathrm{x}} \cdot \vec{\mathrm{x}}} = \sqrt{\mathrm{x}_0^2 + \mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2} \tag{8}$$

By replacing the real numbers $\mathrm{x}_0, \mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3$ for complex numbers:

$$\mathrm{X} = \mathrm{x}_0 + \mathrm{i}\mathrm{y}_0 + \vec{i} \cdot (\vec{\mathrm{x}} + \mathrm{i}\vec{\mathrm{y}}) \tag{9}$$

a *complex* quaternion is obtained [6]. A complex quaternion is also called a *biquaternion*. The imaginary number i, should not be confused with $\vec{i} = (i, j, k)$. In general, a single biquaternion equation

$$(\mathrm{a}_0 + \mathrm{i}\mathrm{b}_0) + \vec{i} \cdot \ [\vec{\mathrm{a}} + \mathrm{i}\vec{\mathrm{b}}] = (\mathrm{c}_0 + \mathrm{i}\mathrm{d}_0) + \vec{i} \cdot \ [\vec{\mathrm{c}} + \mathrm{i}\vec{\mathrm{d}}] \tag{10}$$

is a compact notation of two scalar equations and two vector equations:

$$\begin{aligned} \mathrm{a}_0 &= \mathrm{c}_0 \\ \mathrm{b}_0 &= \mathrm{d}_0 \\ \vec{\mathrm{a}} &= \vec{\mathrm{c}} \\ \vec{\mathrm{b}} &= \vec{\mathrm{d}} \end{aligned} \tag{11}$$

For instance, the four Maxwell equations can be expressed by just one biquaternion equation.

2 Minkowski space and biquaternion operators for physics

The four dimensional Minkowski space can be expressed in biquaternion form:

$$\mathrm{X} = (\mathrm{ict} + \vec{i} \cdot \vec{\mathrm{x}}) \tag{12}$$

where X is the position quaternion. Notice that $\mathrm{x} = 0$, $\vec{\mathrm{y}} = \vec{0}$ and $\mathrm{y} = \mathrm{ct}$. In general a biquaternion represents 8-dimensional space. In case we want to represent 4-dimensional Minkowskian space in quaternions we set $\mathrm{x} = 0$ and $\vec{\mathrm{y}} = \vec{0}$. The lenght of X is invariant under transformation between inertial systems:

$$|\mathrm{X}| = \sqrt{(\mathrm{ict})^2 + \vec{\mathrm{x}} \cdot \vec{\mathrm{x}}} = \sqrt{\mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2 - \mathrm{c}^2\mathrm{t}^2} \tag{13}$$

The biquaternion velocity is defined as:

$$\mathrm{V} = \frac{\mathrm{dX}}{\mathrm{dt}} = \frac{\mathrm{d(ict)}}{\mathrm{dt}} + \vec{i} \cdot \frac{\mathrm{d}\vec{\mathrm{x}}}{\mathrm{dt}} = \mathrm{ic} + \vec{i} \cdot \vec{\mathrm{v}} \tag{14}$$

The length of V is:

$$|\mathrm{V}| = \sqrt{\mathrm{v}^2 - \mathrm{c}^2} = \mathrm{ic}\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}} \tag{15}$$

In stead of using V and dt we might define a relativistic time differential $\mathrm{d}\tau$ (time dilatation):

$$\mathrm{d}\tau = \frac{|\mathrm{V}|}{\mathrm{ic}}\mathrm{dt} = \sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}\mathrm{dt} \tag{16}$$

which is known as the "proper time", and define a "relativistic biquaternion speed":

$$\mathrm{U} = \frac{\mathrm{dX}}{\mathrm{d}\tau} = \frac{\mathrm{ic}}{|\mathrm{V}|}\frac{\mathrm{dX}}{\mathrm{dt}} = \frac{\mathrm{icV}}{|\mathrm{V}|} \tag{17}$$

but then it is obvious that $|\mathrm{U}| = \mathrm{ic}$ and this makes no sense [7]. So either we use the four-dimensional velocity V and the time differential dt, or we use the concept of time-dilatation $\mathrm{d}\tau$ and a three-dimensional velocity $\frac{\mathrm{d}\vec{\mathrm{x}}}{\mathrm{d}\tau}$. We choose for the first option, because it does not make sense to mix 4-dimensional qualities with 3-dimensional qualities in one theory. Now that a physical space has been described, three biquaternion operators for the use of physics are defined as follows:

$$\text{Nabla} \quad : \quad \nabla = \frac{\mathrm{i}}{\mathrm{c}}\frac{\partial}{\partial \mathrm{t}} + \vec{i} \cdot \vec{\nabla}, \qquad \vec{\nabla} = (\frac{\partial}{\partial \mathrm{x}_1}, \frac{\partial}{\partial \mathrm{x}_2}, \frac{\partial}{\partial \mathrm{x}_3}) \tag{18}$$

$$\text{d'Alembert} \quad : \quad \Box = -|\nabla|^2 = -\nabla\nabla^* = \frac{1}{\mathrm{c}^2}\frac{\partial^2}{\partial \mathrm{t}^2} - \vec{\nabla} \cdot \vec{\nabla} \tag{19}$$

3 Electrodynamics in biquaternion form

The quaternion electromagnetic potential and the quaternion current are defined as follows:

$$\mathrm{A} = \frac{\mathrm{i}}{\mathrm{c}}\Phi + \vec{i} \cdot \vec{\mathrm{A}} \tag{20}$$

$$\mathrm{J} = \mathrm{ic}\rho + \vec{i} \cdot \vec{\mathrm{J}} = \mathrm{ic}\rho + \vec{i} \cdot \rho\vec{\mathrm{V}} = \rho\mathrm{V} \tag{21}$$

By applying the quaternion product we can determine the differential of the electromagnetic potential, ∇A:

$$\nabla\mathrm{A} = \left(\frac{i}{c}\frac{\partial}{\partial t} + \vec{i} \cdot \vec{\nabla}\right)\left(\frac{i}{c}\Phi + \vec{i} \cdot \vec{\mathrm{A}}\right) = -\left(\frac{1}{c^2}\frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \vec{A}\right) + \vec{i} \cdot \left[\vec{\nabla} \times \vec{\mathrm{A}} + \frac{\mathrm{i}}{\mathrm{c}}\left(\frac{\partial \vec{\mathrm{A}}}{\partial \mathrm{t}} + \vec{\nabla}\Phi\right)\right] \tag{22}$$

If one defines the electric vector field, magnetic vector field, and an extra scalar field as follows:

$$\vec{\mathrm{E}} = -\frac{\partial \vec{\mathrm{A}}}{\partial \mathrm{t}} - \vec{\nabla}\Phi \tag{23}$$

$$\vec{\mathrm{B}} = \vec{\nabla} \times \vec{\mathrm{A}} \tag{24}$$

$$\mathrm{S} = \frac{1}{c^2}\frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \vec{A} \tag{25}$$

then

$$\nabla \mathrm{A} = -\mathrm{S} + \vec{i} \cdot \left[\vec{\mathrm{B}} - \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{E}}\right] \tag{26}$$

Usually the Maxwell equations are defined by (in biquaternion form):

$$-\nabla^*(\nabla \mathrm{A} + \mathrm{S}) = \mu \mathrm{J} \quad \text{or by} \tag{27}$$

$$\Box \mathrm{A} - \nabla^* \mathrm{S} = \mu \mathrm{J} \quad \text{or by} \tag{28}$$

$$-\nabla^*\left(\vec{i} \cdot \left[\vec{\mathrm{B}} - \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{E}}\right]\right) = \mu \mathrm{J} \tag{29}$$

It is easy to verify that these three equations are identical. Equation (31) can be expanded by applying the quaternion product:

$$\left(-\frac{\mathrm{i}}{\mathrm{c}}\frac{\partial}{\partial \mathrm{t}} + \vec{i} \cdot \vec{\nabla}\right)\left(\vec{i} \cdot \left[\vec{\mathrm{B}} - \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{E}}\right]\right) =$$

$$-\vec{\nabla} \cdot \vec{\mathrm{B}} + \frac{\mathrm{i}}{\mathrm{c}}\vec{\nabla} \cdot \vec{\mathrm{E}} + \vec{i} \cdot \left[\vec{\nabla} \times \vec{\mathrm{B}} - \frac{1}{\mathrm{c}^2}\frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} - \frac{\mathrm{i}}{\mathrm{c}}\left(\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}} + \vec{\nabla} \times \vec{\mathrm{E}}\right)\right] \tag{30}$$

The biquaternion equation is a short hand notation of the famous Maxwell equations in seperated scalar and vector form:

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2 \Phi}{\partial \mathrm{t}^2} - \vec{\nabla}^2\Phi - \frac{\partial}{\partial \mathrm{t}}\left(\frac{1}{\mathrm{c}^2}\frac{\partial \Phi}{\partial \mathrm{t}} + \vec{\nabla} \cdot \vec{\mathrm{A}}\right) = \mathrm{c}^2\mu\rho = \frac{\rho}{\epsilon} \tag{31}$$

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2 \vec{\mathrm{A}}}{\partial \mathrm{t}^2} - \vec{\nabla}^2\vec{\mathrm{A}} + \vec{\nabla}\left(\frac{1}{\mathrm{c}^2}\frac{\partial \Phi}{\partial \mathrm{t}} + \vec{\nabla} \cdot \vec{\mathrm{A}}\right) = \mu\vec{\mathrm{J}} \tag{32}$$

$$\vec{\nabla} \cdot \vec{\mathrm{B}} = 0 \tag{33}$$

$$\vec{\nabla} \cdot \vec{\mathrm{E}} = c^2\mu\rho = \frac{\rho}{\epsilon} \tag{34}$$

$$\vec{\nabla} \times \vec{\mathrm{B}} - \frac{1}{\mathrm{c}^2}\frac{\partial \vec{\mathrm{E}}}{\partial \mathrm{t}} = \mu\vec{J} \tag{35}$$

$$\frac{\partial \vec{\mathrm{B}}}{\partial \mathrm{t}} + \vec{\nabla} \times \vec{\mathrm{E}} = 0 \tag{36}$$

The Lorentz 4-force density biquaternion, $\mathrm{F} = \frac{\mathrm{i}}{\mathrm{c}}\mathrm{P} + \vec{i} \cdot \vec{\mathrm{F}}$, is defined by the following biquaternion equation:

$$\mathrm{F} = \mathrm{J}(\nabla \mathrm{A} + \mathrm{S}) \tag{37}$$

Applying the quaternion product:

$$\begin{aligned} \mathrm{J}(\nabla \mathrm{A} + \mathrm{S}) &= \rho V(\nabla A + S) = \rho(ic + \vec{i} \cdot \vec{\mathrm{V}})\left(\vec{i} \cdot \left[-\frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{E}} + \vec{\mathrm{B}}\right]\right) \\ &= -\vec{\mathrm{J}} \cdot \vec{\mathrm{B}} + \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{J}} \cdot \vec{\mathrm{E}} + \vec{i} \cdot \left[(\rho\vec{\mathrm{E}} + \vec{\mathrm{J}} \times \vec{\mathrm{B}}) + \mathrm{ic}(\rho\vec{\mathrm{B}} - \frac{1}{\mathrm{c}^2}\vec{\mathrm{J}} \times \vec{\mathrm{E}})\right] \end{aligned} \tag{38}$$

In seperated scalar and vector form, the Lorentz force equation is:

$$\begin{aligned} 0 &= \vec{\mathrm{J}} \cdot \vec{\mathrm{B}} && (39) \\ \mathrm{P} &= \vec{\mathrm{J}} \cdot \vec{\mathrm{E}} && (40) \\ \vec{\mathrm{F}} &= \rho\vec{\mathrm{E}} + \vec{\mathrm{J}} \times \vec{\mathrm{B}} && (41) \\ \vec{0} &= \rho\vec{\mathrm{B}} - \frac{1}{\mathrm{c}^2}\vec{\mathrm{J}} \times \vec{\mathrm{E}} && (42) \end{aligned}$$

In the biquaternion Lorentz force equation, J can be eliminated by substituting the biquaternion Maxwell equation:

$$\mathrm{F} = \frac{1}{\mu}\nabla^*(\nabla\mathrm{A} + \mathrm{S})(\nabla\mathrm{A} + \mathrm{S}) \tag{43}$$

By expanding the imaginary scalar part and the real vector part of this equation one finds the well known energy and momentum theorems:

$$\mu(\vec{\mathrm{J}} \cdot \vec{\mathrm{E}}) = -\frac{\partial}{2\partial t}\left[\frac{1}{c^2}E^2 + B^2\right] - \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \tag{44}$$

$$\begin{aligned} \mu\left(\rho\vec{\mathrm{E}} + \vec{\mathrm{J}} \times \vec{\mathrm{B}}\right) &= \left[\frac{1}{c^2}\left((\vec{\nabla} \cdot \vec{E})\vec{E} + (\vec{\nabla} \times \vec{E}) \times \vec{E}\right) + (\vec{\nabla} \times \vec{B}) \times \vec{B}\right] \\ &\quad -\frac{1}{c^2}\frac{\partial(\vec{E} \times \vec{B})}{\partial t} \end{aligned} \tag{45}$$

This demonstrates that electrodynamics can be cast in biquaternion form. By considering a non-zero real scalar and imaginary vector part in the current biquaternion, one can introduce the magnetic monopole and magnetic current. A magnetic 4-current is imaginary with respect to the electric 4-current. The total current can be called an 8-current. One can also consider an 8-potential or an 8-Lorentz force with a non-zero real scalar and imaginary vector part in A or in F. The biquaterion mathematics enables a logical treatment of such extentions of the theory of electrodynamics.

4 A gauge asymmetrical theory of electrodynamics that includes scalar field S

No matter if S=0, which is called the *Lorenz gauge* condition, the biquaternion Maxwell equation is invariant with respect to a gauge transformation of the biquaternion potential:

$$\mathrm{A} \quad \to \quad \mathrm{A}' = \mathrm{A} - \nabla^*\Gamma \tag{46}$$

where Γ is an arbitrary scalar function. The easiest way to prove this is by showing that only the S field is changed by such a gauge transformation:

$$\nabla\mathrm{A} \quad \to \quad \nabla\mathrm{A}' = \nabla\mathrm{A} - \nabla\nabla^*\Gamma = \nabla\ \mathrm{A} + \Box\Gamma \tag{47}$$

and since $\Box\Gamma$ is strictly a scalar function, only the S field is transformed:

$$\mathrm{S} \quad \to \quad \mathrm{S}' = \mathrm{S} + \nabla\nabla^*\Gamma = \mathrm{S} - \Box\Gamma \tag{48}$$

Electrodynamics is gauge invariant, because the expression $(\nabla\mathrm{A} + \mathrm{S})$ in each equation is invariant. It is asssumed that $\vec{\nabla} \cdot \vec{\mathrm{A}}$ can be chosen freely, and that any potential can be gauge transformed into another potential such that the two potentials are not different with respect to measurable physics. Potentials can be transformed into potentials that satisfy a condition, such as the Lorenz condition [8] [9]: S' = 0. The latter requires that $\Box\Gamma = \mathrm{S}$. If we only consider potentials that satisfy the Lorenz condition (S=0) then the gauge transformation Γ has to satisfy $\Box\Gamma = 0$. It is not clear if the wave-nature of Γ has a physical meaning. In case S=0, then the Maxwell equation is defined also by $\Box\mathrm{A} = \mu\mathrm{J}$, which is called the Lorenz inhomogeneous wave equation.

Another type of gauge transformation exists that gives Lorenz's inhomogeneous wave equation:

$$\mu\mathrm{J} \quad \rightarrow \quad \mu\mathrm{J}' \;=\; \mu\mathrm{J} - \nabla^*\Gamma \tag{49}$$

where Γ is an arbitrary scalar field. This is a current gauge transformation. The Maxwell equation is not invariant with respect to this transformation. A regauge of the current J is the special transformation $\Gamma = \mathrm{S}$. After a current regauge, the resulting Maxwell equation automatically has the form of the Lorenz inhomogeneous wave equation in case it is described in terms of the potentials:

$$-\nabla^*(\nabla\mathrm{A} + \mathrm{S}) \;=\; \mu\mathrm{J} - \nabla^*\mathrm{S} \qquad \text{or} \tag{50}$$

$$\Box\mathrm{A} \;=\; \mu\mathrm{J} \qquad \text{or} \tag{51}$$

$$-\nabla^*\left(-\mathrm{S} + \vec{i}\cdot\left[\vec{\mathrm{B}} - \frac{\mathrm{i}}{\mathrm{c}}\vec{\mathrm{E}}\right]\right) \;=\; \mu\mathrm{J} \tag{52}$$

Therefore, solutions of the Maxwell equation are also solutions of the inhomogeneous wave equation. The transformed Maxwell equation also contains scalar field S and is a generalisation of the original Maxwell equation. Rewriting this equation in seperate scalar and vector equations results into the *generalised* Maxwell equations:

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2\Phi}{\partial\mathrm{t}^2} - \vec{\nabla}^2\Phi \;=\; \mathrm{c}^2\mu\rho = \frac{\rho}{\epsilon} \tag{53}$$

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2\vec{\mathrm{A}}}{\partial\mathrm{t}^2} - \vec{\nabla}^2\vec{\mathrm{A}} \;=\; \mu\vec{\mathrm{J}} \tag{54}$$

$$\vec{\nabla}\cdot\vec{\mathrm{E}} + \frac{\partial\mathrm{S}}{\partial\mathrm{t}} \;=\; c^2\mu\rho = \frac{\rho}{\epsilon} \tag{55}$$

$$\vec{\nabla}\cdot\vec{\mathrm{B}} \;=\; 0 \tag{56}$$

$$\frac{\partial\vec{\mathrm{B}}}{\partial\mathrm{t}} + \vec{\nabla}\times\vec{\mathrm{E}} \;=\; 0 \tag{57}$$

$$\vec{\nabla}\times\vec{\mathrm{B}} - \frac{1}{\mathrm{c}^2}\frac{\partial\vec{\mathrm{E}}}{\partial\mathrm{t}} - \vec{\nabla}\mathrm{S} \;=\; \mu\vec{J} \tag{58}$$

Analogous to the addition of the displacement current, which allowed Maxwell to derive the homogeneous field wave equations, the addition of the scalar field related charge and current terms allow for the derivation of the inhomogeneous potential wave equations without the Lorenz gauge. Because the potential gauge freedom is lost, one should regard the expression S as a real and physical field. It is doubtful if potential gauge freedom exists, which means that $\nabla\cdot\vec{\mathrm{A}}$ has an arbitrary value, considering the fact that S can be deduced from the technical specifications of electrodynamic equipment. We assume it is possible that S can be measured, just like the electric or magnetic field. Next, it is shown that a natural current gauge must satisfy the condition $\Box\mathrm{S}_\mathrm{g} = 0$. First, the gauge transform of the partial derivative of current is described by:

$$\nabla(\mu\mathrm{J}) \quad \rightarrow \quad \nabla(\mu\mathrm{J}') \;=\; \nabla(\mu\mathrm{J}) - \nabla\nabla^*\mathrm{S}_\mathrm{g} = \nabla(\mu\mathrm{J}) + \Box\mathrm{S}_\mathrm{g} \tag{59}$$

Since $\Box S_g$ is strictly a scalar, only the scalar part of $\nabla(\mu J)$ is transformed:

$$\mu(\frac{\partial \rho'}{\partial t} + \vec{\nabla} \cdot \vec{J}') = \mu(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}) + \Box S_g \tag{60}$$

Because charge is conserved, the scalar part of $\nabla(\mu J)$ and of $\nabla(\mu J')$ is zero:

$$\mu(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J}) = \mu(\frac{\partial \rho'}{\partial t} + \vec{\nabla} \cdot \vec{J}') = 0 \tag{61}$$

And therefore $\Box S_g = 0$. Hence, a natural regauge is possible only if $\Box S = \Box S_g = 0$. The current gauge condition has a straightforward and physical interpretation: the conservation of charge. If an electrodynamical system has a natural tendency to satisfy the Lorenz inhomogeneous wave equation, then this can be described by a conditional current regauge, such that charge conservation is not violated. By assuming that the scalar is zero everywhere (S=0) the original Maxwell equations are found. The condition S=0 is not a free-to-choose gauge condition, but it simply is a physical condition. A physical interpretation of the current gauge transform is the following.

Within the framework of quantum electrodynamics, charge polarizations are a possibility in vacuum, and this means that the charge density and current density in the vacuum can fluctuate. A current gauge transformation is the classical equivalent of this concept. A regauge of the biquaternion current can be regarded as the change in vacuum charge-current density, such that the potentials associated with both real and "virtual" charge-current, are solutions of the Lorenz inhomogeneous wave equation. During a current regauge the microscopic "virtual" charge-current fluctuations are forced by the presence of a real charge-current to form an orderly pattern on a larger scale. The current regauge is therefore a physical process, in stead of a pure mathematical transformation, and we suggest that the entropy of the current regauge process is negative.

4.1 Vacuum field wave solutions

Assuming the current gauge condition is true, the partial derivatives of the left and right hand side of the Maxwell equation are gauge invariant. After current regauging, this equation is:

$$\nabla \Box A = -\nabla\nabla\nabla^* A = -\nabla\nabla^*\nabla A = \Box \left(-S + \vec{i} \cdot \left[-\frac{i}{c}\vec{E} + \vec{B} \right] \right) = \mu \nabla J \tag{62}$$

and reformulated into seperate scalar and vector wave equations:

$$\frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} - \vec{\nabla}^2 S = \mu \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} \right) = 0 \tag{63}$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \vec{\nabla}^2 \vec{E} = \mu \left(-\frac{\partial \vec{J}}{\partial t} - c^2 \vec{\nabla} \rho \right) \tag{64}$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \vec{\nabla}^2 \vec{B} = \mu \left(\vec{\nabla} \times \vec{J} \right) \tag{65}$$

In case J=0 the latter equations become the homogeneous field wave equations. It is well known that, for S=0 and J=0, a transversal electromagnetic wave is a natural solution of the Maxwell equations. Now suppose that $\vec{B} = 0$ and J=0. From the generalised Maxwell equations the following wave equations can be deduced:

$$\frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} - \vec{\nabla}^2 S = 0 \tag{66}$$

$$\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \vec{\nabla}^2\vec{E} = 0 \quad (67)$$

that have the solutions:

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad (68)$$

$$S = S_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad (69)$$

$$\frac{\omega}{c^2}\vec{E} = \vec{k}S \quad (70)$$

$$\vec{k}\cdot\vec{E} = \omega S \quad (71)$$

This is a longitudinal electroscalar wave (LES wave). The possible existence of such a wave might be the subject of experimentation, and therefore our theory is testable. Associated with a LES wave is an energy flow density vector $\vec{E}S$. In the next sections it is shown that this energy flow density vector is part of an extended energy theorem. A more general set of vacuum wave equations can be found via the following equation

$$\vec{\nabla}S + \frac{k}{c^2}\frac{\partial\vec{E}}{\partial t} = 0 \quad (72)$$

After combining the extended Maxwell equations with this extra field equation, we find the following vacuum wave equations

$$\vec{\nabla}\cdot\vec{\nabla}S - \frac{k}{c^2}\frac{\partial^2 S}{\partial t^2} = 0 \quad (73)$$

$$\vec{\nabla}\vec{\nabla}\cdot\vec{E} - \frac{k}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad (74)$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} + \frac{(1-k)}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad (75)$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} + \frac{(1-k)}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad (76)$$

$$(77)$$

These equations are valid wave equations only if $k \in [0,1]$ and have superluminal solutions for $k \in < 0,1 >$. The longitudinal part of the electric field interacts with the S field, forming a LES wave with speed $\frac{c}{\sqrt{k}}$. The transversal part of the electric field interacts with the B field, forming a TEM wave with speed $\frac{c}{\sqrt{1-k}}$. For $k = \frac{1}{2}$ both waves have speed $c\sqrt{2}$. For k=0 the TEM wave solution is the usual luminal TEM wave with speed c, and the LES wave speed is infinite. This means there might be a simultaneous coexistance of retarded and immediate action at-a-distance. Vice verse, for k=1 the speed of the LES wave is c, and the speed of the TEM wave is infinite, and this might be the classical equivalent of tunneling photons. Such photons are accompanied by LES waves that have phase velocity of c, according to this theory. If one of the wave types has infinite speed then the other wave type has speed c, and this adds new meaning to the value c. In [15] a superluminal microwave with average speed of 4.7c was measured by G. Nimtz. This value is close to $\sqrt{22}c$, and this suggests a value of $k = \frac{21}{22}$ and a LES wave speed of $\sqrt{\frac{22}{21}}c \approx c$. G.F. Ignatiev [16] measured a signal speed of 1.12c, and this value is close to $\sqrt{\frac{5}{4}}c$. This suggest a value of $k = \frac{1}{5}$ and a LES wave with speed $\sqrt{5}c \approx 2.24c$.

4.2 The generalised Lorentz force

A generalised Lorentz force definition is expressed by the following equation:

$$\mathrm{F} = \mathrm{J}\nabla\mathrm{A} \tag{78}$$

In expression $\mathrm{J}\nabla\mathrm{A}$ the scalar field S is no longer cancelled, while in $\mathrm{J}(\nabla\mathrm{A} + \mathrm{S})$ in classical Electrodynamics the S field is cancelled. This is similar to the generalisation of the Maxwell equation. In the next section it is shown that this generalisation of the Lorentz force is also based on the current regauge transformation. Rewriting the latter biquaternion equation into separate scalar and vector equations:

$$\begin{aligned} \vec{\mathrm{J}} \cdot \vec{\mathrm{B}} &= 0 && (79) \\ \vec{\mathrm{J}} \cdot \vec{\mathrm{E}} - \rho \mathrm{c}^2 \mathrm{S} &= \mathrm{P} && (80) \\ \rho\vec{\mathrm{E}} + \vec{\mathrm{J}} \times \vec{\mathrm{B}} - \vec{\mathrm{J}}\mathrm{S} &= \vec{\mathrm{F}} && (81) \\ \vec{\mathrm{B}} - \frac{\vec{\mathrm{v}}}{\mathrm{c}^2} \times \vec{\mathrm{E}} &= 0 && (82) \end{aligned}$$

The first equation shows two power flow terms: $\vec{\mathrm{J}} \cdot \vec{\mathrm{E}}$ and $\rho\mathrm{c}^2\mathrm{S}$. The first term is the electrical energy flow that is usually associated with a current $\vec{\mathrm{J}}$ in a wire, and that compensates for the dissipated energy. The second power flow term is new and has to be associated with a 'static' charge. It is like the Zero Point Energy exchange between a charge and the vacuum. The energy flows inwards and outwards with respect to the volume of the charge. The fourth equation defines the vector Lorentz force and it contains an extra term $\vec{\mathrm{J}}\mathrm{S}$. This term is similar to a radiation reaction force, but it is independent of the acceleration of the charge.

It is also interesting to examine the special case of $\mathrm{S}\vec{\mathrm{v}} = \vec{\mathrm{E}}$. For this case the Lorentz force quaternion reduces to:

$$\begin{aligned} \rho(\mathrm{v}^2 - \mathrm{c}^2)\mathrm{S} &= \mathrm{P} && (83) \\ \vec{0} &= \vec{\mathrm{F}} && (84) \end{aligned}$$

This means that a charge can exchange energy with an external electromagnetic potential due to the S-field, despite of the absence of an electromagnetic force. The speed of the particle is constant in this situation. If S and $\vec{\mathrm{E}}$ are also wave solutions then the relation $\mathrm{S}\vec{\mathrm{v}} = \vec{\mathrm{E}}$ shows a longitudinal electroscalar wave with speed $\vec{\mathrm{v}}$. The energy exchange diminishes with $\mathrm{v}^2\mathrm{S}$ and becomes zero when v approaches c. In [14] an alternative description of the De Brogly wave is given, based on a new principle of *intrinsic* (belonging to the particle) potential energy of electromagnetic origin. The wave nature of a particle is in essence a periodic transformation of kinetic energy into intrinsic potential energy, and vice verse. The intrinsic potential energy of a particle might as well be electroscalar in nature. In other words: the LES wave with $\mathrm{S}\vec{\mathrm{v}} = \vec{\mathrm{E}}$ might be a new description of the De Brogly wave if we consider the S field and the E field as intrinsic to the particle.

4.3 Generalised energy and momentum theorems

Within the generalised Lorentz force equation $\mathrm{J}\nabla\mathrm{A} = \mathrm{F}$ we can substitute for J its definition in terms of potentials, $\frac{1}{\mu}\Box A$. Then we get:

$$\Box\mathrm{A}\nabla\mathrm{A} = \mu\mathrm{F} \tag{85}$$

When we evaluate the imaginary scalar part of this equation in terms of fields and sources, we get the following energy equation:

$$\mu(\vec{\mathrm{E}} \cdot \vec{\mathrm{J}} - \rho\mathrm{c}^2\mathrm{S}) = -\frac{\partial}{2\partial\mathrm{t}}\left[\mathrm{S}^2 + \frac{1}{\mathrm{c}^2}\mathrm{E}^2 + \mathrm{B}^2\right] - \vec{\nabla} \cdot (\vec{\mathrm{E}} \times \vec{\mathrm{B}} + \vec{\mathrm{E}}\mathrm{S}) \tag{86}$$

The term $\vec{E}S$ represents an extra power flux vector that can be associated with the longitudinal electroscalar wave. The Poynting vector $\vec{E}\times\vec{B}$ is usually associated with the transversal electromagnetic wave. The energy term S^2 is the energy stored in the field S. The real vector part of the biquaternion equation is:

$$\mu\left(\rho\vec{E}+\vec{J}\times\vec{B}-\vec{J}S\right) = \left[\frac{1}{c^2}\left((\vec{\nabla}\cdot\vec{E})\vec{E}+(\vec{\nabla}\times\vec{E})\times\vec{E}\right)+(\vec{\nabla}\times\vec{B})\times\vec{B}\right]+ \left[S\vec{\nabla}S-\vec{\nabla}\times(S\vec{B})\right]+\frac{1}{c^2}\frac{\partial(\vec{E}S-\vec{E}\times\vec{B})}{\partial t} \tag{87}$$

This equation is the extended momentum theorem in the generalised Maxwell theory. Usually, Maxwell's stress tensor represents the terms between the first pair of square brackets. It can be generalised, such that it represents also the terms between the second pair of square brackets. The power flow terms of both TEM waves and LES waves are present also in this equation.

These equations can be derived also by applying the current regauge to the original energy and momentum theorems:

$$\begin{aligned}\vec{J}\cdot\vec{E} \quad\rightarrow\quad & \left(\vec{J}+\frac{1}{\mu}\vec{\nabla}S\right)\cdot\vec{E} = \\ & \vec{J}\cdot\vec{E}+\frac{1}{\mu}(\vec{\nabla}S)\cdot\vec{E} = \\ & \vec{J}\cdot\vec{E}+\frac{1}{\mu}\vec{\nabla}\cdot(S\vec{E})-S\vec{\nabla}\cdot\vec{E} = \\ & \vec{J}\cdot\vec{E}+\frac{1}{\mu}\left(\vec{\nabla}\cdot(S\vec{E})-S\vec{\nabla}\cdot\vec{E}\right) = \\ & \vec{J}\cdot\vec{E}+\frac{1}{\mu}\left(\vec{\nabla}\cdot(S\vec{E})-S\left(\frac{\rho}{\epsilon}-\frac{\partial S}{\partial t}\right)\right) = \\ & \left(\vec{J}\cdot\vec{E}-Sc^2\rho\right)+\frac{1}{\mu}\left(\vec{\nabla}\cdot(S\vec{E})+\frac{\partial(S^2)}{2\partial t}\right)\end{aligned} \tag{88}$$

$$\begin{aligned}\rho\vec{E}+\vec{J}\times\vec{B} \quad\rightarrow\quad & \left(\rho-\epsilon\frac{\partial S}{\partial t}\right)\vec{E}+\left(\vec{J}+\frac{1}{\mu}\vec{\nabla}S\right)\times\vec{B} = \\ & \left(\rho\vec{E}+\vec{J}\times\vec{B}\right)-\epsilon\frac{\partial S}{\partial t}\vec{E}+\frac{1}{\mu}\vec{\nabla}S\times\vec{B} = \\ & \left(\rho\vec{E}+\vec{J}\times\vec{B}\right)-\epsilon\frac{\partial(\vec{E}S)}{\partial t}+\frac{1}{\mu}\vec{\nabla}\times(\vec{B}S)+\frac{S}{\mu}\left(\frac{\partial\vec{E}}{c^2\partial t}-\vec{\nabla}\times\vec{B}\right) = \\ & \left(\rho\vec{E}+\vec{J}\times\vec{B}\right)-\epsilon\frac{\partial(\vec{E}S)}{\partial t}+\frac{1}{\mu}\vec{\nabla}\times(\vec{B}S)+S\left(-\vec{J}-\frac{1}{\mu}\vec{\nabla}S\right) = \\ & \left(\rho\vec{E}+\vec{J}\times\vec{B}-S\vec{J}\right)+\frac{1}{\mu}\left(-\frac{\partial(\vec{E}S)}{c^2\partial t}+\vec{\nabla}\times(\vec{B}S)-S\vec{\nabla}S\right)\end{aligned} \tag{89}$$

This shows that the current regauge not only changes the biquaternion Lorentz force, but also introduces extra terms with respect to the field energy, radiation energy flow and the stress tensor.

4.4 The source of scalar field S

Including the extra scalar field in the Maxwell equations rises the question how to induce this field. Keeping in mind that the potentials always satisfy the Lorenz inhomogeneous wave equations after the current regauge, and that the retarded potentials are solutions of these wave equations, it is necessary to verify if a scalar field can be derived from the retarded potentials:

$$\Phi(\vec{x},t) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{x}',t'_{ret})}{|\vec{x}-\vec{x}'|}d^3x' \quad (90)$$

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{j}(\vec{x}',t'_{ret})}{|\vec{x}-\vec{x}'|}d^3x' \quad (91)$$

$$t'_{ret} = t - \frac{|\vec{x}-\vec{x}'|}{c} \quad (92)$$

This can be done by Fourier analysis. The Fourier transformed retarded potentials are defined by:

$$\Phi(\vec{x},\omega) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' \quad (93)$$

$$\vec{A}(\vec{x},\omega) = \frac{\mu_0}{4\pi}\int_V \vec{j}(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' \quad (94)$$

with

$$\rho(\vec{x}',\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\rho(\vec{x}',t)e^{i\omega t}dt \quad (95)$$

$$\vec{j}(\vec{x}',\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{j}(\vec{x}',t)e^{i\omega t}dt \quad (96)$$

The Fourier transformed scalar field is defined as

$$\begin{aligned} S(\vec{x},\omega) &= \frac{1}{c^2}\frac{\partial\Phi(\vec{x},\omega)}{\partial t} + \vec{\nabla}\cdot\vec{A}(\vec{x},\omega) \\ &= -i\omega\frac{1}{4\pi c^2\epsilon_0}\int_V \rho(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' + \vec{\nabla}\cdot\left(\frac{\mu_0}{4\pi}\int_V \vec{j}(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x'\right) \\ &= -i\omega\frac{\mu_0}{4\pi}\int_V \rho(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' + \frac{\vec{\mu_0}}{4\pi}\int_V \vec{j}(\vec{x}',\omega)\cdot\nabla\left(\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\right)d^3x' \end{aligned} \quad (97)$$

By using the Fourier transform of the continuity equation

$$\vec{\nabla}'\cdot\vec{j}(\vec{x}',\omega) - i\omega\rho(\vec{x}',\omega) = 0 \quad (98)$$

it is possible to evaluate further the first integral:

$$\begin{aligned} -i\omega\frac{\mu_0}{4\pi}\int_V \rho(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' &= -\frac{\mu_0}{4\pi}\int_V \left(\vec{\nabla}'\cdot\vec{j}(\vec{x}',\omega)\right)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x' \\ &= -\frac{\mu_0}{4\pi}\int_V \vec{\nabla}'\cdot\left(\vec{j}(\vec{x}',\omega)\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\right)d^3x' \\ &\quad +\frac{\vec{\mu_0}}{4\pi}\int_V \vec{j}(\vec{x}',\omega)\cdot\nabla'\left(\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\right)d^3x' \\ &= \frac{\vec{\mu_0}}{4\pi}\int_V \vec{j}(\vec{x}',\omega)\cdot\nabla'\left(\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\right)d^3x' \end{aligned} \quad (99)$$

In the last step Gauss's theorem was applied: the first integral term vanishes if $\vec{j}(\vec{x}', \omega)$ is assumed to be limited and tends to zero at large distances. The final expression for the scalar field in frequency domain is:

$$\begin{aligned} S(\vec{x}, \omega) &= \frac{\vec{\mu_0}}{4\pi} \int_V \vec{j}(\vec{x}', \omega) \cdot \nabla' \left(\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x} - \vec{x}'|} \right) d^3x' + \frac{\vec{\mu_0}}{4\pi} \int_V \vec{j}(\vec{x}', \omega) \cdot \nabla \left(\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x} - \vec{x}'|} \right) d^3x' \\ &= 0 \end{aligned} \tag{100}$$

The inverse Fourier transform of this expression gives us the expression for the S field in time domain: $S(\vec{x}, t) = 0$. In a similar way one can derive the same result for the advanced potentials. Also in case that S is derived from the Liénard-Wiechert potentials, which are a special case retarded potentials, one finds that S=0. However, the retarded and advanced potentials are not the most general solutions of the Lorenz inhomogeneous wave equations. An interesting special case for getting a non zero S is by setting $\vec{E} = \vec{0}$ and $\vec{B} = \vec{0}$. In this case the field equations are

$$\frac{\partial S}{\partial t} = \frac{\rho}{\epsilon} \tag{101}$$

$$-\vec{\nabla} S = \mu \vec{J} \tag{102}$$

Adding the gradient of the first equation to the time differential of the second equation gives

$$0 = \frac{1}{\epsilon} \vec{\nabla} \rho + \mu \frac{\partial \vec{J}}{\partial t} \tag{103}$$

The divergence of this equation (and using the continuity of charge equation) finally gives

$$0 = \vec{\nabla} \cdot \vec{\nabla} \rho - \frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} \tag{104}$$

which is a charge density wave. Also in this case the scalar field is a wave, but directly associated with a charge density wave. In classical electrodynamics there cannot be sources without the presence of an electric or magnetic field. In this theory it is possible to have sources that only induce a scalar field. This example shows that the scalar field can be sourced directly by a dynamic charge density distribution.

5 Conclusions

It is possible to describe classical electrodynamics in the form of two biquaternion equations. This form is very useful in order to generalise electrodynamics. Generalising the Maxwell equation by introducing an extra scalar field is comparable with Maxwell's introduction of the displacement current that allowed for the derivation of the homogeneous field wave equations. This theory predicts the existance of longitudinal electroscalar waves in vacuum. Such a wave might be used to transmit and receive signals. The power density vector of LES waves is $\vec{E}S$, thus energetic and wireless signals might be transmittable in LES wave form and received at a distance.

6 Acknowledgement

The authors are grateful to V. Onoochin, H. Puthoff and M. Ibison for valuable discussions and scientific comments.

References

[1] W.R. Hamilton, *On a new Species of Imaginary Quantities connected with a theory of Quaternions.* Proceedings of the Royal Irish Academy 2 (13 November 1843) 424-434

[2] J.C. Maxwell, *A Treatise on Electricity & Magnetism,* , (1893) Dover Publications, New York ISBN 0-486-60636-8 (Vol. 1) & 0-486-60637-6 (Vol. 2)

[3] D. Sweetser and G. Sandri, *Maxwells vision: Electromagnetism with Hameltons quaternions,* http://www.emis.de/proceedings/QSMP99/sweeters.pdf

[4] A. Waser, *Quaternions in electrodynamics,* http://www.aw-verlag.ch/Documents/QuaternionsInElectrodynamicsEN02.pdf

[5] J.W. Marshall, *Quaternions as 4-Vectors,* American Journal of Physics 24 (1956) 515-522

[6] R. Fueter, Comm. Math. Helv., 7, 307, (1934-35), 8, 371, (1936-37)

[7] Y. Yong-Gwan, *On the Nature of Relativistic Phenomena,* Apeiron 6 Nr.3-4 (July-Oct.1999)

[8] L.V. Lorenz, *Über die Identität der Schwingungen des Lichts mit den elektrischen Strömen,* Poggendorfs Annalen der Physik, Bd. 131, 1867.

[9] B. Riemann, *Schwere, Elektrizität und Magnetismus* (1861),K. Hattendorf, Ed. Hannover, Germany: Carl Rümpler, 1880.

[10] A. Liénard, L' Éclairage Électique 165 (1898); E. Wiechert, Ann. Phys. 4 676 (1901).

[11] L.D. Landau and E.M. Lifshitz, Teoria Polia (Nauka, Moscow, 1973) [English translation: *The Classical Theory of Fields* (Pergamon, Oxford, 1975)].

[12] A.E. Chubykalo and S.J. Vlaev, *Double (implicit and explicit) dependence of the electromagnetic field of an accelerated charge on time: Mathematical and physical analysis of the problem* , http://xxx.lanl.gov/abs/physics/9803037

[13] D.G. Boulware, *Radiation from a Uniformly Accelerated Charge* , Annals of Physics 124, 169 (1980).

[14] W.A. Hofer, *Beyond Uncertainty: the internal structure of electrons and photons* , http://xxx.lanl.gov/abs/quant-ph/9611009, TU Wien, November 1996

[15] G. Nimtz, *Superluminal signal velocity,* http://xxx.lanl.gov/abs/physics/9812053

[16] G. F. Ignatiev and V. A. Leus, *On a superluminal transmission at the phase velocities,* http://www.truth.myweb.nl/Art-p16.pdf